Deltoid *

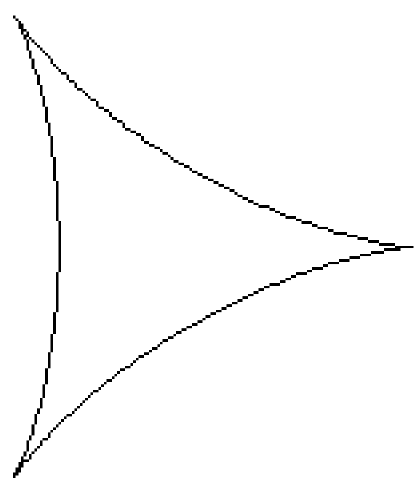
The Deltoid curve was conceived by Euler in 1745 in connection with his study of caustics.

Its parametric formula is

$$x = 2\cos(t) + \cos(2t)$$
$$y = 2\sin(t) - \sin(2t)$$
$$0 < t \leq 2\pi,$$

and its implicit equation is:

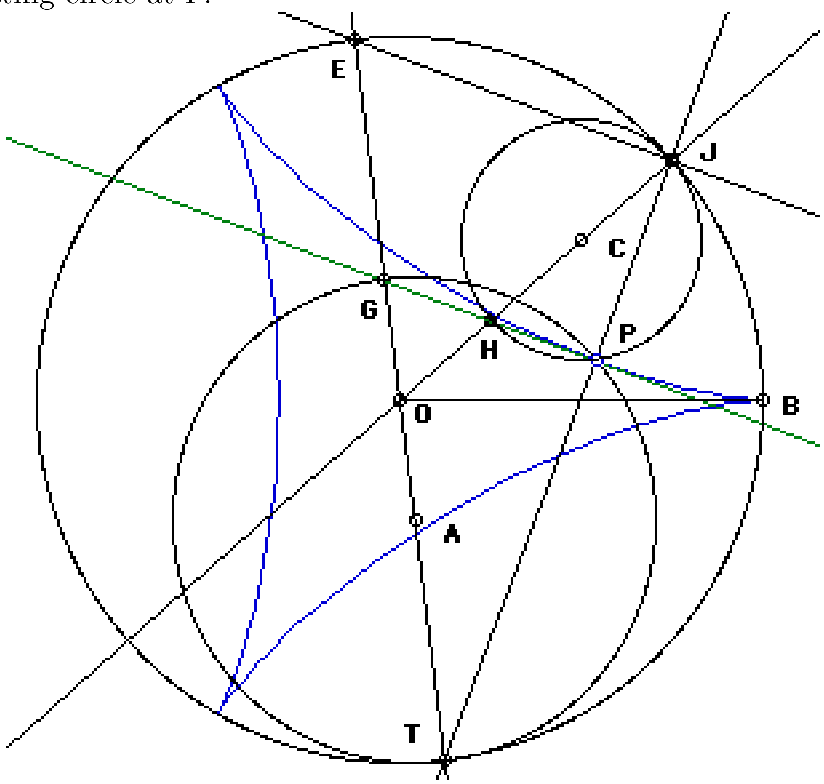
$$(x^2 + y^2)^2 - 8x(x^2 - 3y^2) + 18(x^2 + y^2) - 27 = 0$$



The Deltoid or Tricuspid

*This file is from the 3D-XploreMath project.
Please see http://rsp.math.brandeis.edu/3D-XplorMath/index.html

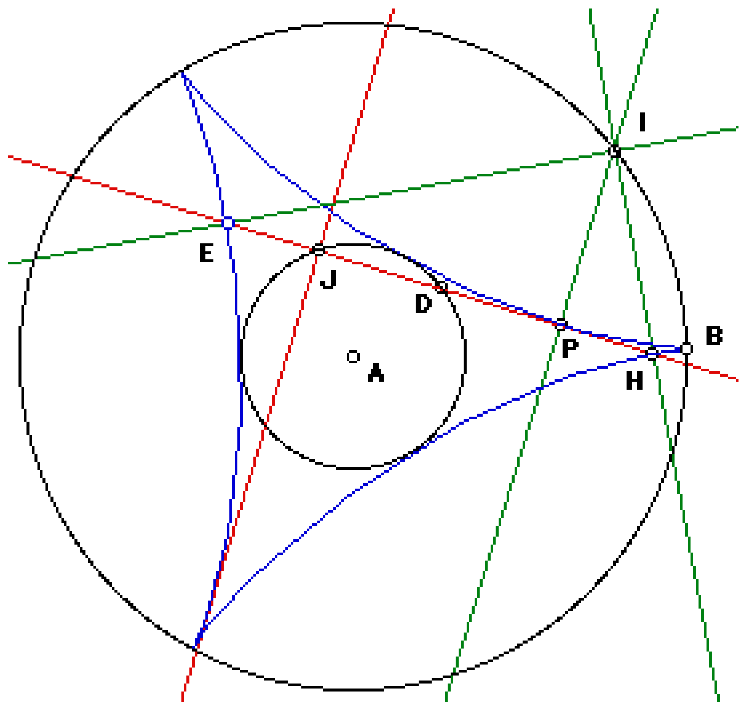
The Deltoid is also known as the Tricuspid, and can be defined as the trace of a point on one circle that rolls inside another circle of 3 or 3/2 times as large a radius. The latter is called double generation. The figure below shows both of these methods. O is the center of the fixed circle of radius a, C the center of the rolling circle of radius $a/3$, and P the tracing point. OHCJ, JPT and TAOGE are colinear, where G and A are distant $a/3$ from O, and A is the center of the rolling circle with radius $2a/3$. PHG is colinear and gives the tangent at P. Triangles TEJ, TGP, and JHP are all similar and $TP/JP = 2$. Angle JCP = 3∗Angle BOJ. Let the point Q (not shown) be the intersection of JE and the circle centered on C. Points Q, P are symmetric with respect to point C. The intersection of OQ, PJ forms the center of osculating circle at P.



The Deltoid has numerous interesting properties.

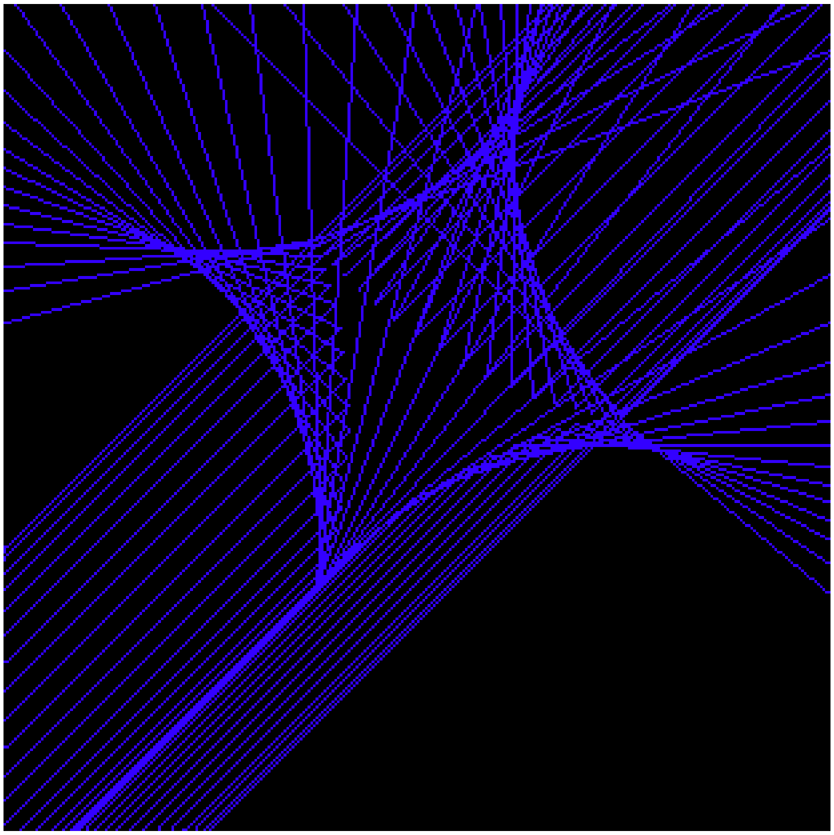
1 Properties

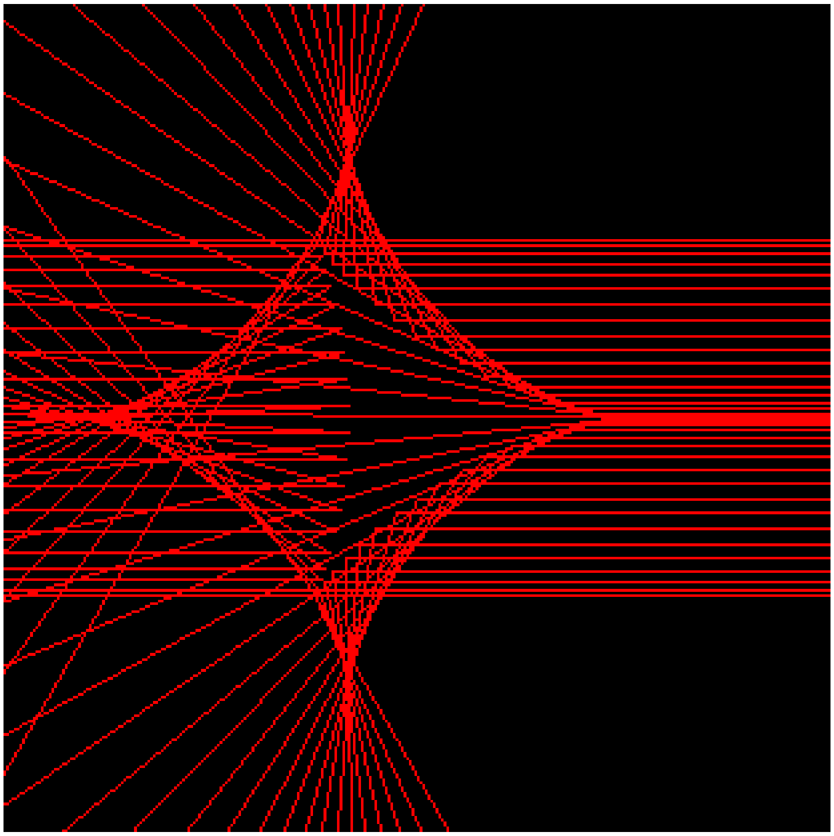
1.1 Tangent



Let A be the center of the curve, B be one of the cusp points,and P be any point on the curve. Let E, H be the intersections of the curve and the tangent at P. The segment EH has constant length distance[E,H]== 4/3*distance[A,B]. The locus of midpoint D of the tangent segment EH is the inscribed circle. The normals at E,P,H are concurrent, and the locus of these intersections is the circumscribed circle. If J is the intersection of another tangent, cutting EH at right angle, then the locus traced by J (the Deltoid's orthoptic) is the inscribed circle.

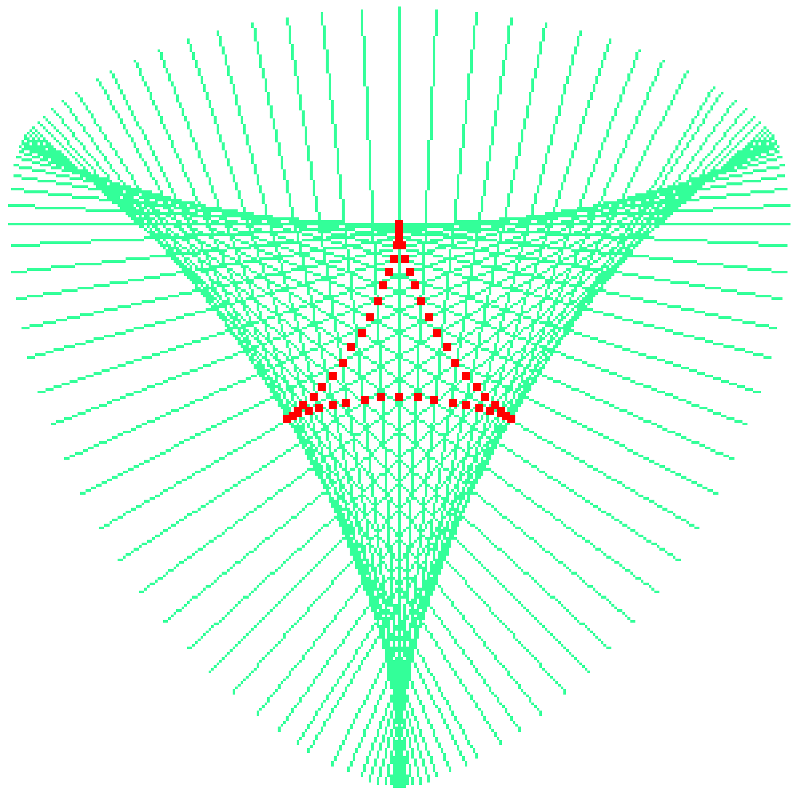
1.2 The Deltoid and the Astroid





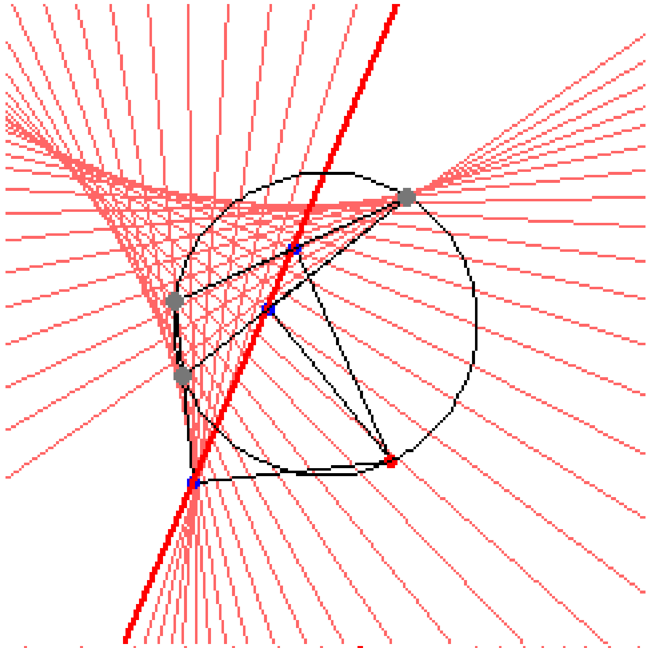
The caustic of the Deltoid with respect to parallel rays in any direction is an Astroid.

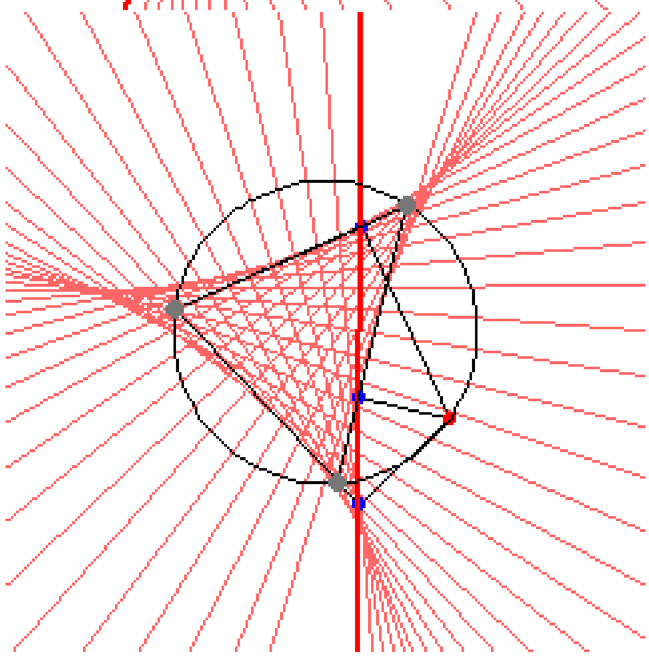
1.3 Evolute



The evolute of Deltoid is another Deltoid. (In fact, the evolutes of all epicycloids and hypocycloids are scaled version of themselves.) In the above figure, the evolute is shown as the envelope of its normals.

1.4 Simson Lines



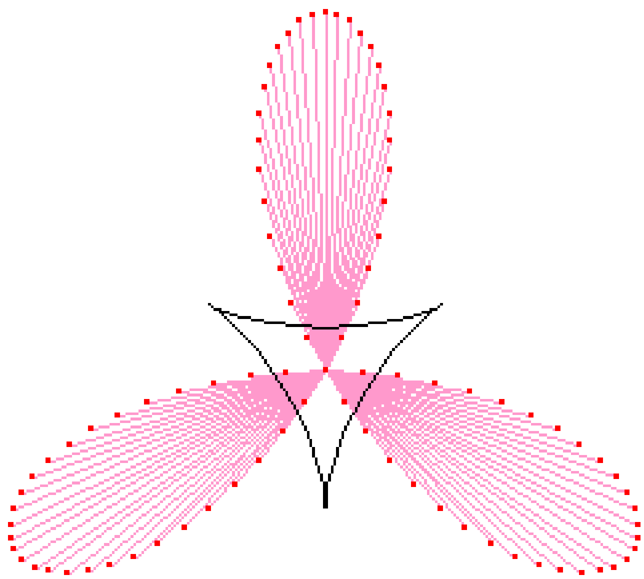


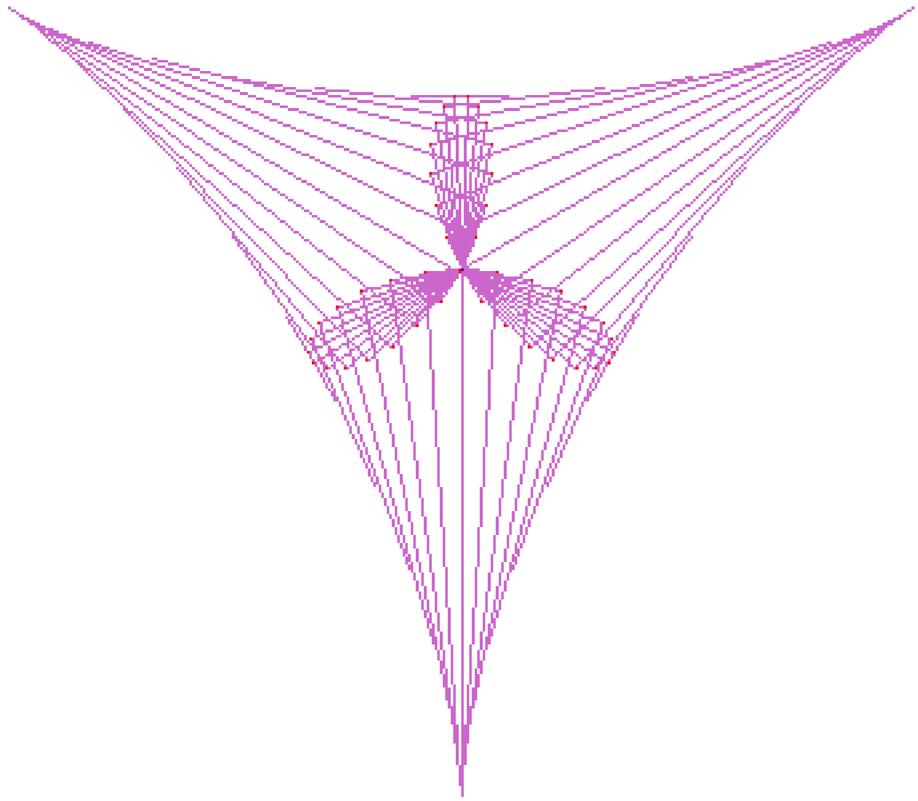
The Deltoid is the envelope of the Simson lines of any triangle. (Robert Simson, 1687–1768)

Step by step description:

1. Let a triangle be inscribed in a circle. 2. Pick any point P on the circle. 3. Mark a point Q1 on any side of the triangle such that line[P,Q1] is perpendicular to it, extending the side if necessary. 4. Similarly, find points Q2 and Q3 with respect to P for the other two sides. 5. The points Q1, Q2, and Q3 are colinear, and the line passing through them is called the Simson line of the triangle with respect to P. 6. Find Simson lines for the other points P on the circle. Their envelope is the deltoid. Amazingly, this is true for any triangle.

1.5 Pedal, Radial, and Rose





The pedal curve of a Deltoid with respect to a cusp, vertex, or center is a folium curve with one, two, or three loops respectively. The last one is called the trifolium, a three petalled rose. The Deltoid's radial is a trifolium too.

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